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Influence of Anisotropy on Thermosolutal Convection in Porous Media with Magnetic Field Effect

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Abstract: In the present study, both anisotropy and magnetic field effects on bi-diffusive natural convection in a rectangular cavity filled with a porous medium saturated by a binary fluid are investigated analytically for fully developed flow regime. The cavity is heated isothermally by the sides and its horizontal walls are thermally insulated or conducted. The porous medium is anisotropic in permeability whose principal axes are oriented in a direction that is arbitrary to the gravity field. On the basis of the generalized Brinkman-extended Darcy model of newtonian fluids on steady flow through porous media, analytical expressions were obtained for the flow and thermal fields, the concentration of speaces, the average Nusselt and Sherwood numbers in terms of the Darcy number, the anisotropic permeability ratio, the orientation angle of the principal axes and the Hartmann number. The limiting case corresponding to pure porous media ($Da \rightarrow 0$) and pure fluid media ($Da \rightarrow \infty$) for the thermal conditions mentioned on the cavity completed these results in order to compare them to those obtained in the literature. It is found that, Nusselt and Sherwood numbers increase by increasing anisotropic parameters of the porous medium while increasing magnetic field magnitude greatly reduces the intensity of the flow and thus affects significantly heat and mass transfer.

Keywords: Porous Medium, Thermosolutal Convection, Magnetic Field, Anisotropy

1. Introduction

In recent decades, diffusive double convection has received much attention from researchers. This interest is due to the multiple practical applications in various fields (thermal treatments of polluted soils; storage of radioactive waste; solar ponds; oceanography; geophysics, etc.). The diffusion of pollutants represents an interesting application of bi-diffusive convection. Indeed, from a local source of pollutant in the aquifer medium, the effects of the dispersion due to the average speed of the flow due to the thermal gradients tend to disperse the polluting agent through the porous layer and can thus affect the water bodies [1]. To this end, in an aquifer environment, convective movements contribute by homogenizing effects to the diffusion contaminants [2]. In this process, the soil, the interface between the various media, constitutes the porous medium, the seat of the transfer of heat and mass (pollutant particle) to these hydro-systems by the phenomenon of convection. Interest in double-diffusive convection in porous media began after Nield [3] investigated the stability of a horizontal porous layer, heated and salty from below, as well as a very detailed summary of the work carried out in the past is presented in the book by Nield and Bejan [4]. The problem of natural thermosolutal convection in a horizontal and porous anisotropic cavity has been the subject of some research works. The Nusselt number is maximum when the Darcy number is sufficiently large according to Zheng and all [5]. Smail BENISSAAD and all [6] have shown that the control parameters greatly influence flow, heat transfer and mass. Akowanou in 2007 [7] studied convective transfer in porous cavities subjected to a transverse magnetic field, it appears that the application of a magnetic field considerably reduces the speed of the flow. In this study, we will look at the heat and mass transfer induced by thermosolutal convection in an anisotropic horizontal porous cavity in permeability and under the effect of the transverse external magnetic field. Abbas ATTIA [13] studied the suppression of thermosolutal instabilities by the action of a magnetic field with the geometry used a rectangular enclosure filled with an aqueous solution. The aim is to study the influence of the magnetic field and anisotropy on convective flow and the transfer of heat and mass.

2. Nomenclature

A	geometric aspect ratio, L'/H'
a, b, c	constants equation, (13)
B	applied magnetic field
g	gravity acceleration
H '	cavity height, m
Ha	Hartmann number
J'	current density
K	thermal conductivity
K_1, K_2	permeabilities along the main axes
$\overline{\overline{K}}$	Tensor (second order) of permeability
Nu	Nusselt number
Sh	Sherwood number
P	dimensionless pressure, $(P'-P'_0)H'^2/\rho_0\,\alpha_p^2$
Le	Lewis number, α_p/D
K*	Permeability anisotropy ratio
R_T	Rayleigh Darcy number- thermal
C	dimensionless concentration, $(C'-C_0)\,/\Delta C^*$
q′	constant heat flow (per unit area), $W.m^{-2}$
ΔC^*	Characteristic concentration, $j'H'/D_p$;
ΔT^*	Characteristic temperature, $q'H'/k_p$
t	dimensionless time, $t'\alpha_p\,/H'^2$
N	Ratio of volume forces, $\beta_c\Delta C^*/\ \beta_T\Delta T^*$
L	cavity length
T	dimensionless temperature, $(T'-T_0)/\Delta T^*$
(u, v)	dimensionless velocities in the directions Ox et Oy, $(u'H'/\alpha_p,\ v'H'/\alpha_p)$
(x, y)	Cartesian coordinates adimensional, (x'/H', y'/H')
L_x	length of the central region of the cavity
Greek symbols	
α_p	thermal diffusivity of the saturated porous medium $k_p/(\rho C)$
(ρc)	heat capacity, $W.K^{-1}$
β_T	coefficient of thermal expansion, K^{-1} β_c solutal expansion coefficient, kg.mol. L^{-1}
ε	dimensionless porosity, ε'/σ
ϑ	kinematic viscosity of the fluid, $m^2.s^{-1}$
σ	heat capacity ratio $(\rho C)_p/(\rho C)_f$
Indices and exhibitors	
0	reference state
t	Thermal
c	Solutale
p	Porous
f	Fluid

3. Geometry and Mathematical Formulation

We consider a rectangular cavity of height H "and length L". Vertical walls are subjected to uniform flows of heat and space while horizontal walls are adiabatic and waterproof. This enclosure is filled with a porous medium saturated with a binary fluid.

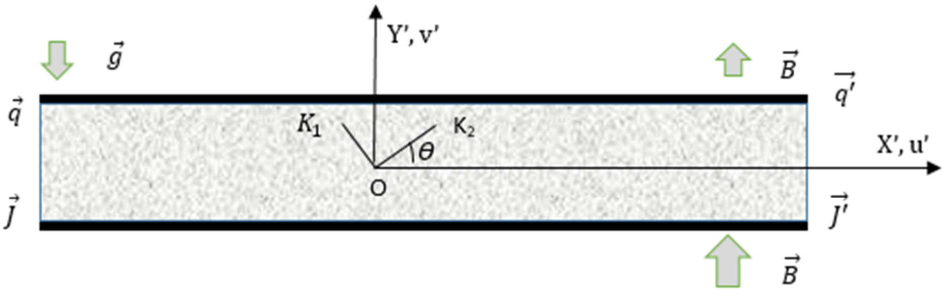

Figure 1. *Physical model and coordinate axes.*

The regime considered here is the steady state with flow developed in the porous channels. The equations of continuity, motion, energy and concentration are written:

$$\nabla\overrightarrow{V'} = 0 \tag{1}$$

$$V' = \frac{\overline{\overline{K}}}{\mu}\left[\nabla\overrightarrow{P'} + \rho'\vec{g} + \mu_{eff}\nabla^2\overrightarrow{V'} + \vec{J}\wedge\vec{B}\right] \tag{2}$$

$$\nabla\overrightarrow{J'} = 0;\ \vec{J} = \gamma\left(-\nabla\emptyset + \vec{V}'\vec{B}\right) \tag{3}$$

$$\left(\rho C_p\right)_m\frac{\partial T\prime}{\partial t} + \left(\rho C_p\right)_f\nabla.\left(\overrightarrow{V'}T'\right) = k\nabla^2 T' \tag{4}$$

$$\boldsymbol{\varepsilon}\frac{\partial T\prime}{\partial t} + \nabla.\left(\overrightarrow{V'}C'\right) = k\nabla^2(DC') \tag{5}$$

The second order permeability tensor $\overline{\overline{K}}$ of the porous medium in the axis system of Figure 1 is written:

$$\overline{\overline{K}} = \begin{bmatrix} K_1\cos\theta^2 + K_2\sin\theta^2 & (K_2 - K_1)\sin\theta\cos\theta \\ (K_2 + K_1)\sin\theta\cos\theta & K_2\cos\theta^2 + K_1\sin\theta^2 \end{bmatrix} \tag{6}$$

Since the porous medium is electrically isolated, the electric field will be zero everywhere from where

$$\nabla\emptyset = 0 \tag{7}$$

$$\text{So we have } \overrightarrow{J'} = \gamma\left(\overrightarrow{V'}\wedge\vec{B}\right) \tag{8}$$

Considering (u',v') the coordinates of the filtration speed $\overrightarrow{V'}$ following (ox',oy')

$$\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} = 0 \tag{9}$$

$$au' - cv' = \frac{K_1}{\mu}\left(-\frac{\partial P'}{\partial x'} + \mu_{eff}\nabla^2\overrightarrow{u'} - \gamma u'B^2\right) \tag{10}$$

$$-cv' + bv' = \frac{K_1}{\mu}\left(-\frac{\partial P'}{\partial y'} + \mu_{eff}\nabla^2\overrightarrow{v'} - \rho\vec{g}\right) \tag{11}$$

$$\sigma\frac{\partial T\prime}{\partial t\prime} + u'\frac{\partial T\prime}{\partial x\prime} + v'\frac{\partial T'}{\partial y'} = \alpha\left(\frac{\partial^2 T'}{\partial x'^2} + \frac{\partial^2 T'}{\partial y'^2}\right) \tag{12}$$

$$\varepsilon\frac{\partial C\prime}{\partial t\prime} + u'\frac{\partial C\prime}{\partial x\prime} + v'\frac{\partial C'}{\partial y'} = D\left(\frac{\partial^2 C'}{\partial x'^2} + \frac{\partial^2 C'}{\partial y'^2}\right) \tag{13}$$

By deriving equations (10) and (11) and by subtracting (11) from equation (10) we eliminate the term of the pressure and we obtain:

$$a\frac{\partial u\prime}{\partial x'} - b\frac{\partial v'}{\partial y'} + c\left(\frac{\partial u'}{\partial x'} - \frac{\partial v'}{\partial y'}\right) = \frac{K_1}{\mu}\left[\mu_{eff}\left(\frac{\partial}{\partial y'}(\nabla^2 u') - \frac{\partial}{\partial x'}(\nabla^2 v')\right) - \gamma B^2\frac{\partial u'}{\partial y'} - \rho_0 g\beta_T\frac{\partial T'}{\partial x'} - \rho_0 g\beta_C\frac{\partial C'}{\partial x'}\right] \tag{14}$$

With

$$\rho = \rho_0[1 - \beta_T(T' - T_0) - \beta_C(C' - C_0)] \tag{15}$$

By considering the Boussinesq approximation, we therefore have:

$$a\frac{\partial u'}{\partial x'} - b\frac{\partial v'}{\partial y'} + c\left(\frac{\partial u'}{\partial x'} - \frac{\partial v'}{\partial y'}\right) = \frac{K_1}{\mu}\left[-\gamma B^2\frac{\partial u'}{\partial y'} - \rho_0 g\left(\beta_T\frac{\partial T'}{\partial x'} + \beta_C\frac{\partial C'}{\partial x'}\right) + \mu_{eff}\left(\frac{\partial^3 u'}{\partial y'^3} - \frac{\partial^3 v'}{\partial x'^3} + \frac{\partial}{\partial x'\partial y'}\left(\frac{\partial u'}{\partial x'} - \frac{\partial v'}{\partial y'}\right)\right]\quad (16)$$

In steady state we have:

$$\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} = 0 \quad (17)$$

$$u'\frac{\partial T'}{\partial x'} + v'\frac{\partial T'}{\partial y'} = \alpha\left(\frac{\partial^2 T'}{\partial x'^2} + \frac{\partial^2 T'}{\partial y'^2}\right) \quad (18)$$

$$u'\frac{\partial C'}{\partial x'} + v'\frac{\partial C'}{\partial y'} = D\left(\frac{\partial^2 C'}{\partial x'^2} + \frac{\partial^2 C'}{\partial y'^2}\right) \quad (19)$$

$\sigma = \frac{(\rho C_p)_m}{(\rho C_p)_f}$ is the ratio of heat capacities.

$$\alpha = \frac{K}{(\rho C_p)_f} \text{ the diffusivity of the porous medium.} \quad (20)$$

D The Fick mass diffusivity of the porous medium.

3.1. Conditions to the Limits

3.1.1. On Vertical Walls

$$x' = -\frac{L'}{2};\, v' = 0;\, T' = T_1 \text{ et } C' = C_1 \quad (21)$$

$$x' = \frac{L'}{2};\, v' = 0;\, T' = T_2 \text{ et } C' = C_2$$

3.1.2. On Horizontal Walls

$$y' = \mp\frac{H'}{2};\, u' = 0;\, \frac{\partial T'}{\partial x'} = 0 \text{ et } \frac{\partial C'}{\partial x'} = 0 \quad (22)$$

$$y' = \mp\frac{H'}{2};\, u' = 0;\, T' = \frac{\Delta T}{L}x' \text{ et } C' = \frac{\Delta C}{L}x'$$

3.2. Dimensionalization

The dimensionless variables are written

$$\begin{cases} (x,y) = \frac{(x',y')}{H'} \quad C = \frac{C'-C_0}{\Delta C'} \\ (u,v) = \frac{(u',v')H'}{\alpha_f} \quad \Delta C' = \frac{J'H'}{D} \\ T = \frac{T'-T_0}{\Delta T'} \quad \Delta T' = \frac{q'H'}{k} \\ T = \frac{T'-T_0}{\Delta T'} \end{cases} \quad (23)$$

Equations (16), (17), (18) and (19) are written

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (24)$$

$$a\frac{\partial u}{\partial y} - b\frac{\partial v}{\partial x} + c\left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right) = \frac{K_1}{\mu}\left[-\frac{\gamma B^2\alpha}{H^2}\frac{\partial u}{\partial y} - \frac{\rho_0 g\beta_T\Delta T}{H}\frac{\partial T}{\partial x} - \frac{\rho_0 g\beta_C\Delta C K_1}{H}\frac{\partial C}{\partial x} + \frac{\mu_{eff.\alpha}}{H^4}\left(\frac{\partial^3 u}{\partial y^3} - \frac{\partial^3 v}{\partial x^3} + \frac{\partial^2}{\partial x\partial y}\left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right)\right)\right] \quad (25)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \quad (26)$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = \frac{1}{Le}\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right) \quad (27)$$

On a so

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{28}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{29}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=\frac{1}{Le}\left(\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}\right) \tag{30}$$

$$(a+Ha^2)\frac{\partial u}{\partial y}-b\frac{\partial v}{\partial x}+c\left(\frac{\partial u}{\partial x}-\frac{\partial v}{\partial y}\right)=Ra_H\left(\frac{\partial T}{\partial x}+N\frac{\partial C}{\partial x}\right)+\lambda Da\left[\frac{\partial^3 u}{\partial y^3}-\frac{\partial^3 v}{\partial x^3}+\frac{\partial^2}{\partial x\partial y}\left(\frac{\partial u}{\partial x}-\frac{\partial v}{\partial y}\right)\right] \tag{31}$$

With $Ha=B(K_1\gamma/\mu)^{\frac{1}{2}}$ is the Hartmann number, $Ra_H=K_1 g\beta H\Delta T/\alpha\mu$ is the Rayleigh number based on the height of the cavity, $Da=K_1/H^2$ is Darcy's number; and $\lambda={}^{\mu_{eff}}/_{\mu}$ is the relative viscosity. In practice, the approximation $\mu_{eff}\approx\mu$ is often used. Therefore, λ will be taken equal to unity.

In addition, equations (21), (22) and (23) associated with equations (28), (29), (30), (31) are written:

* *On vertical walls*

$$x=\frac{-1}{2A}=-\frac{1}{2\delta}, v=0; T=0\ et\ C=0 \tag{32}$$

$$x=\frac{1}{2A}=\frac{1}{2\delta}, v=0; T=1\ et\ C=1 \tag{33}$$

4. Analytical Solution of the Equations

In this part, the differential equations governing the phenomenon studied and subjected to the specified boundary conditions will be solved analytically.

In this part, let's take a look at the central region of the cavity

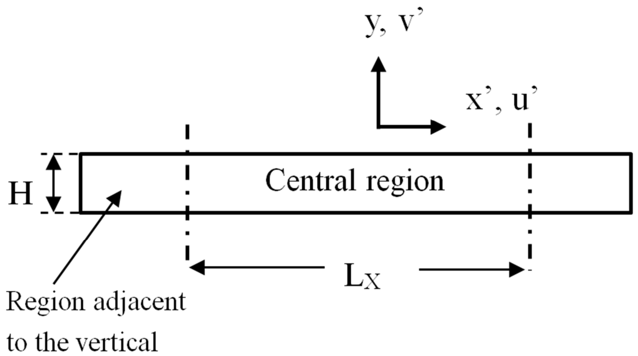

Figure 2. Cavity of great extension and characteristic scale of the variables x and y.

L_x et H are the characteristic scale of the variables x and y in the central region of the cavity

$$L_x\sim x;\ H\sim y \tag{34}$$

According to (34) the scale analysis, on the basis of (28), we can write:

$$\frac{u}{L_x}\sim\frac{v}{H}\ \text{so}\ \frac{u}{v}\sim\frac{L_x}{H}$$

since $\frac{L}{H}\gg\frac{L_x}{H}$ and $A=\frac{H}{L}\ll 1$; then $\frac{u}{v}>>1$.

Thus we can conclude that the flow in the central region of the cavity is in the horizontal direction ox'. This result has been validated by Cormack et Coll [8] and Vasseur et Coll. [9]. Consequently, the component u of the flow velocity in the central region depends only on the y-coordinate:

$$u=u(y) \tag{35}$$

From equation (E28a) we deduce that the transverse speed v is zero

$$v=0 \tag{36}$$

The temperature and concentration in the central region of the cavity depend only on the ordinate

$$T=T(y) \tag{37}$$

$$C=C(y) \tag{38}$$

Under the same conditions of application of the thermal gradient on the walls of a horizontal cavity containing a fluid medium, Birth [10] being the phenomenon of convection by thermo-capillarity, has shown in the past that the thermal gradient along of the horizontal axis ox 'is constant. Thus, following the proposal made by this author, we write:

$$\frac{\partial T'}{\partial x'}=\frac{\Delta T}{L}\ \text{and}\ \frac{\partial C'}{\partial x'}=\frac{\Delta C}{L} \tag{39}$$

By combining (E23) with (E35) we therefore have:

$$\frac{\partial T}{\partial x}=A\ \text{and}\ \frac{\partial C}{\partial x}=\delta \tag{40}$$

Based on considerations (35), (36), (37), (38) and (40) we have:

$$\frac{-(a+Ha^2)}{\lambda Da}\frac{du}{dy}+\frac{d^3u}{dy^3}=\frac{Ra_H(A+N\delta)}{\lambda Da} \tag{41}$$

$$\frac{d^2T}{dy^2}=Au \tag{42}$$

$$\frac{d^2C}{dy^2}=Le\delta u \tag{43}$$

4.1. Flow Velocity

Taking $\varepsilon^2=\frac{a+Ha^2}{Da}$; $\mu_{eff}=\mu$ et $\lambda=1$, the equation (41) becomes

$$\frac{d^3u}{dy^3}+\varepsilon^2\frac{du}{dy}=\frac{Ra_H(A+N\delta)}{Da} \tag{44}$$

Solving the equation (44), taking into account the boundary conditions and those based on the centro-symmetry of the flow field, the velocity is written:

$$u = \frac{Ra_H(A+N\delta)}{\varepsilon^2 Da}\left[\frac{\sinh(\varepsilon y)}{2\sinh\left(\frac{\varepsilon}{2}\right)} - y\right] \tag{45}$$

In the following two limited cases of interest will be investigated, one of which will take into account low porosity media $Da \to 0$ (pure porous medium) and the other medium with high porosity $Da \to \infty$ (pure porous medium) and the other medium with high porosity;

Case of pure fluid medium ($Da \gg 1$)

For $Da \to \infty$ then $\varepsilon \to 0$, (45) becomes:

$$u = \frac{Ra_H(A+N\delta)}{\varepsilon^2 Da}\left[(24-\varepsilon^2)\left(\frac{1}{6}y + \frac{\varepsilon^2}{36}y^3 + \frac{1}{720}y^5\right) - y\right] \tag{46}$$

Case of the pure porous medium ($Da \ll 1$)

For $Da \to 0$ then $\varepsilon \to \infty$, (45) becomes:

$$u = \frac{Ra_H(A+N\delta)}{\varepsilon^2 Da}\left[\frac{1}{2}\exp\varepsilon\left(y-\frac{1}{2}\right) - y\right] \tag{47}$$

Equations (46) and (47) are consistent with those obtained by Garandet and Alboussière [11] as well as Akowanou [12] for not only small Hartmann numbers but also for $\delta = 0$.

4.2. Temperature and Concentration Fields

Horizontal adiabatic walls

Considering equations (39), (45); solving (42) with the boundary conditions of (33) we have:

$$T = \frac{Ra_H(A+N\delta)}{\varepsilon^2 Da}\left[\frac{\sinh(\varepsilon y)}{2\sinh\left(\frac{\varepsilon}{2}\right)} - \frac{1}{6}y^3 + y\left(\frac{1}{8} - \frac{cotanh\left(\frac{\varepsilon}{2}\right)}{2\varepsilon}\right)\right] + Ax \tag{48}$$

$$C = \frac{Ra_H(A+N\delta)}{\varepsilon^2 Da}\left[\frac{\sinh(\varepsilon y)}{2\sinh\left(\frac{\varepsilon}{2}\right)} - \frac{1}{6}y^3 + y\left(\frac{1}{8} - \frac{cotanh\left(\frac{\varepsilon}{2}\right)}{2\varepsilon}\right)\right] + \delta x \tag{49}$$

a) Case of the pure porous medium (Da very small and ε very large)

$$T = \frac{Ra_H(A+N\delta)}{\varepsilon^2 Da}\left[\frac{1}{2\varepsilon^2}\exp\varepsilon\left(y-\frac{1}{2}\right) - \frac{1}{6}y^3 + y\left(\frac{1}{8} - \frac{1}{2\varepsilon}\right)\right] + Ax \tag{50}$$

$$T = \frac{Ra_H(A+N\delta)}{\varepsilon^2 Da}\left[\frac{1}{2\varepsilon^2}\exp\varepsilon\left(y-\frac{1}{2}\right) - \frac{1}{6}y^3 + y\left(\frac{1}{8} - \frac{1}{2\varepsilon}\right)\right] + \delta x \tag{51}$$

b) Case of the pure fluid medium (Da very large and ε tends towards 0)

$$T = \frac{Ra_H(A+N\delta)}{\varepsilon^2 Da}\left[\left(\frac{-1}{24} + \frac{\varepsilon^2}{720} - \frac{\varepsilon^4}{23040}\right)y + \left(\frac{7\varepsilon^4}{34560} - \frac{\varepsilon^2}{144}\right)y^3 + \frac{\varepsilon^2}{2880}(24-\varepsilon^2)y^5\right] + Ax \tag{52}$$

$$T = \frac{Ra_H(A+N\delta)}{\varepsilon^2 Da}\left[\left(\frac{-1}{24} + \frac{\varepsilon^2}{720} - \frac{\varepsilon^4}{23040}\right)y + \left(\frac{7\varepsilon^4}{34560} - \frac{\varepsilon^2}{144}\right)y^3 + \frac{\varepsilon^2}{2880}(24-\varepsilon^2)y^5\right] + \delta x \tag{53}$$

Equation (52) is the expression found by Akowanou [12] in the case of non-binary fluids ($\delta = 0$)

Thermally conductive horizontal walls

Considering equations (40), (45); solving (42) with the boundary conditions of (32) and (33) we have:

$$T = \frac{Ra_H(A+N\delta)}{\varepsilon^2 Da}\left[\frac{\sinh(\varepsilon y)}{2\varepsilon^2\sinh\left(\frac{\varepsilon}{2}\right)} - \frac{1}{6}y^3 + y\left(\frac{1}{24} - \frac{1}{\varepsilon^2}\right)\right] + Ax \tag{54}$$

$$C = \frac{Ra_H(A+N\delta)}{\varepsilon^2 Da}\left[\frac{\sinh(\varepsilon y)}{2\varepsilon^2\sinh\left(\frac{\varepsilon}{2}\right)} - \frac{1}{6}y^3 + y\left(\frac{1}{24} - \frac{1}{\varepsilon^2}\right)\right] + \delta x \tag{55}$$

c) Case of the pure porous medium (Da very small and ε very large)

$$T = \frac{Ra_H(A+N\delta)}{\varepsilon^2 Da}\left[\frac{1}{2\varepsilon^2}\exp\varepsilon\left(y-\frac{1}{2}\right) - \frac{1}{6}y^3 + y\left(\frac{1}{24} - \frac{1}{\varepsilon^2}\right)\right] + Ax \tag{56}$$

$$C = \frac{Ra_H(A+N\delta)}{\varepsilon^2 Da}\left[\frac{1}{2\varepsilon^2}\exp\varepsilon\left(y-\frac{1}{2}\right) - \frac{1}{6}y^3 + y\left(\frac{1}{24} - \frac{1}{\varepsilon^2}\right)\right] + \delta x \tag{57}$$

d) Case of the pure fluid medium (Da very large and ε tends towards 0)

$$T = \frac{Ra_H(A+N\delta)}{\varepsilon^2 Da}\left[\left(\frac{3}{\varepsilon^3} - \frac{1}{8}\right)y + \varepsilon^2\left(\frac{1}{2\varepsilon^2} - \frac{1}{36}\right)y^3 + \left(\frac{\varepsilon^2}{30} - \frac{\varepsilon^4}{720}\right)y^5\right] + Ax \tag{58}$$

$$C = \frac{Ra_H(A+N\delta)}{\varepsilon^2 Da}\left[\left(\frac{3}{\varepsilon^3} - \frac{1}{8}\right)y + \varepsilon^2\left(\frac{1}{2\varepsilon^2} - \frac{1}{36}\right)y^3 + \left(\frac{\varepsilon^2}{30} - \frac{\varepsilon^4}{720}\right)y^5\right] + \delta x \tag{59}$$

Equation (58) is validated by the results of the work of Akowanou [7]; Grandet and Alboussière [11] for low intensity fields and for a fluid containing almost no solute ($\delta = 0$).

The expression (40) is identical to that obtained by Vasseur and Hasnaoui [9] in the limit of small Hartmann numbers.

5. Heat Transfer rate

The average Nusselt number Nu_H is defined as the ratio of the total amount of heat transferred by convection to that received by conduction through the porous cavity. This number not only measures the porous layer but also is evaluated on the straight vertical hot wall, we have:

$$Nu_H = \frac{Q_{convectif}}{Q_{conductif}} \tag{60}$$

$$Q_{convectif} = \int_{-H/2}^{H/2}\left[\left(k\frac{\partial T'}{\partial x'}\right) - \left(\rho C_p\right)_f u'T'\right]dy' \tag{61}$$

$$Q_{conductif} = \frac{kH\Delta T}{L} \tag{62}$$

In dimensionless variables the transfer rate is written

$$Nu_H = \frac{1}{A}\int_{-1/2}^{1/2}\left(\frac{\partial T}{\partial x} - uT\right)dy \tag{63}$$

Horizontal adiabatic walls

$$Nu_H = 1 + \left(\frac{Ra_H(A+\delta N)}{\varepsilon^2 Da}\right)^2\left\{\frac{1}{120} - \frac{1}{24\varepsilon}cotanh\left(\frac{\varepsilon}{2}\right) - \frac{2}{\varepsilon^4} + \frac{1}{\varepsilon\sinh\left(\frac{\varepsilon}{2}\right)}[-\frac{\sinh(\varepsilon)-\varepsilon}{8\varepsilon^2\sinh\left(\frac{\varepsilon}{2}\right)} + \left(\frac{1}{2\varepsilon^2} - \frac{1}{24}\right)\cosh\left(\frac{\varepsilon}{2}\right) + \frac{1}{4\varepsilon}cotanh\left(\frac{\varepsilon}{2}\right)\cosh\left(\frac{\varepsilon}{2}\right)]\right\} \tag{64}$$

Thermally conductive horizontal walls

$$Nu_H = 1 + \left(\frac{Ra_H(A+\delta N)}{\varepsilon^2 Da}\right)^2\left[\frac{1}{720} - \frac{1}{\varepsilon^2}\left(\frac{3}{\varepsilon^2} + \frac{1}{6}\right) + \frac{3cotanh\left(\frac{\varepsilon}{2}\right)}{2\varepsilon^3} - \frac{\sinh(\varepsilon)-\varepsilon}{8\varepsilon^3\sinh\left(\frac{\varepsilon}{2}\right)^2}\right] \tag{65}$$

6. The Mass Transfer Rate

The average Sherwood count $\overline{Sh}$ is the dimensionless number to characterize mass transfers in a fluid and an interface. It is defined by the ratio of mass transfer by convection to mass transfer by diffusion (Thomas Kilgore Sherwood).

$$Sh = \frac{J_{convection}}{J_{diffusion}} \tag{66}$$

Considering

$$\overline{Sh} = \int_0^1\left(Sc * uC - \frac{\partial C}{\partial x}\right)_{x=cte}dy \tag{67}$$

And

$$Sh = \int_{-H/2}^{H/2}\overline{Sh}\,dy \tag{68}$$

So we have

$$Sh = \int_{-1/2}^{1/2}\left(uC - \frac{\partial C}{\partial x}\right)dy \tag{69}$$

Horizontal adiabatic walls

$$Sh = 1 + \left(\frac{Ra_H(A+\delta N)}{\varepsilon^2 Da}\right)^2[\frac{1}{120} - \frac{2}{\varepsilon^4} + \frac{1}{24\varepsilon}cotanh\left(\frac{\varepsilon}{2}\right) - \frac{1}{\varepsilon\sinh\left(\frac{\varepsilon}{2}\right)}(\frac{\sinh(\varepsilon)-\varepsilon}{8\varepsilon^2\sinh\left(\frac{\varepsilon}{2}\right)} + \left(\frac{1}{8} - \frac{1}{2\varepsilon^2}\right)\cosh\left(\frac{\varepsilon}{2}\right) - \frac{1}{4\varepsilon}cotanh\left(\frac{\varepsilon}{2}\right)\cosh\left(\frac{\varepsilon}{2}\right))] \tag{70}$$

Thermally conductive horizontal walls

$$Sh = 1 + \left(\frac{Ra_H(A+\delta N)}{\varepsilon^2 Da}\right)^2\left[\frac{1}{720} - \frac{1}{\varepsilon^2}\left(\frac{3}{\varepsilon^2} + \frac{1}{6}\right) + \frac{3cotanh\left(\frac{\varepsilon}{2}\right)}{2\varepsilon^3} - \frac{\sinh(\varepsilon)-\varepsilon}{8\varepsilon^2\sinh\left(\frac{\varepsilon}{2}\right)}\right] \tag{71}$$

7. Result and Discussion

The influence of the characteristic parameters namely: Ha (Hartmann number), K^* (anisotropy ratio), Θ (angle of inclination of the main axes of the porous medium), Ra (Rayleigh number), Da (Darcy number) and A (geometric aspect ratio of the cavity) on the thermosolutal flow have been illustrated by the following figures.

Figure 3 shows the distribution of the velocity in the central region of the porous medium for $Da = 0.01$; a=1; $Ra = 100$ and $A = 0.5$.

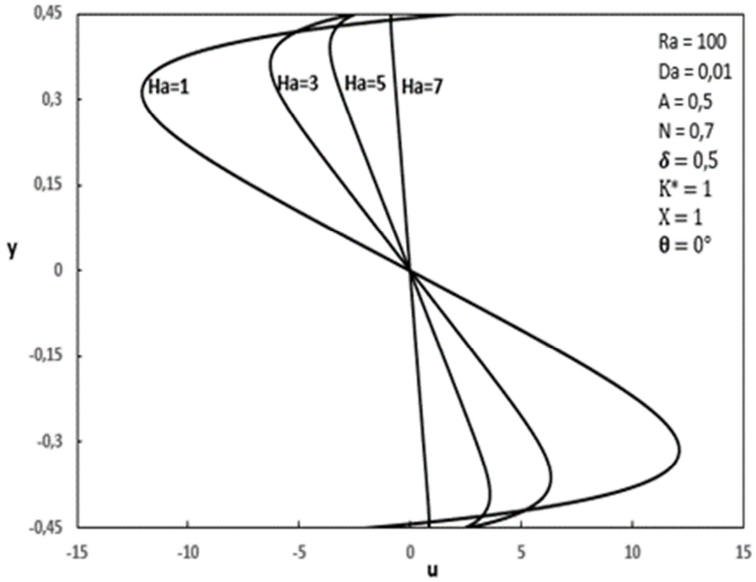

Figure 3. *Velocity in the central region for different values of Ha.*

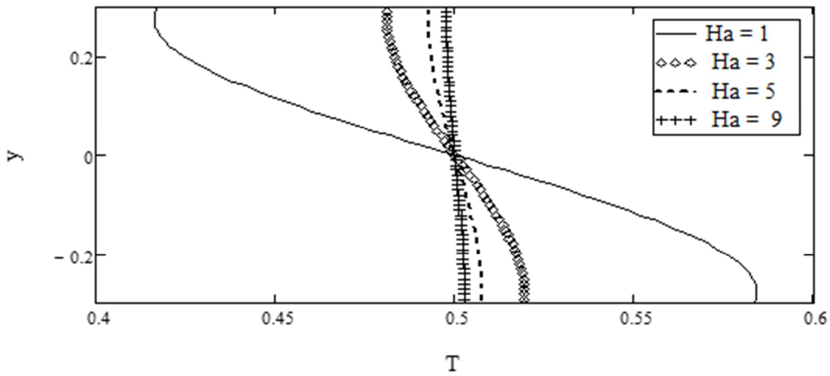

Figure 4. *Influence of Ha on temperature in the central region.*

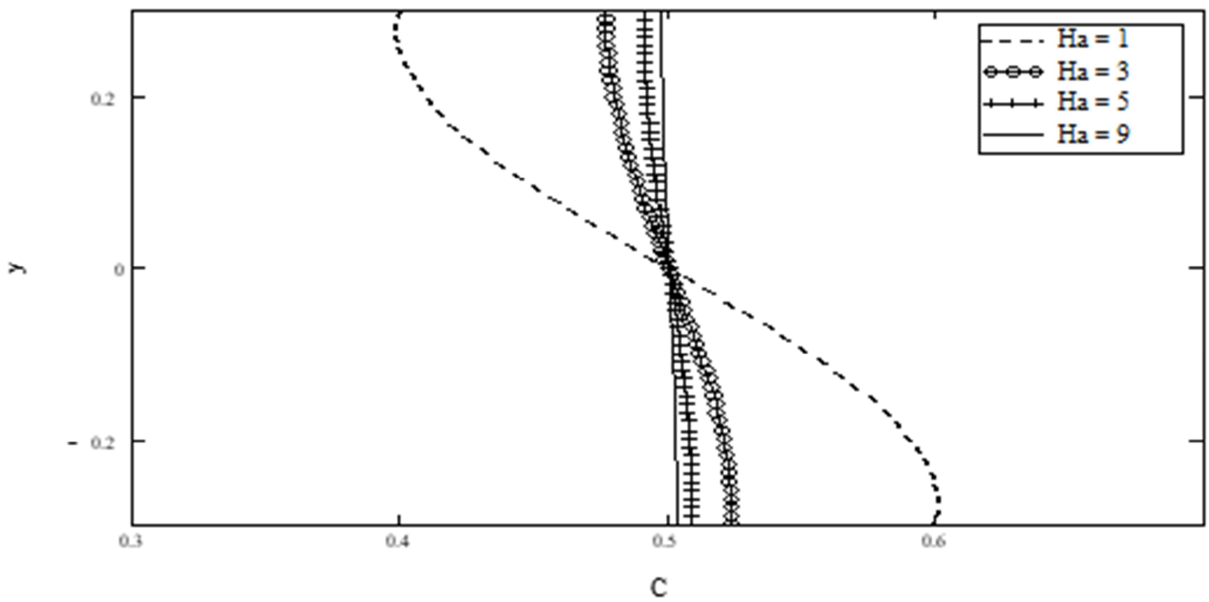

Figure 5. *Influence of Ha on concentration in the central region.*

From the analysis of Figures 4 and 5, we first deduce that the temperature and concentration profiles have symmetry at their level with respect to the central axis of the porous medium. Second, the more the Hartmann number changes, the more the temperature and concentration profiles gradually decrease. Finally, we observe for all the curves, they merge with their respective tangent at the points of contact with the limiting horizontal walls of the porous medium. This reflects the fact that these walls were imposed thermally insulated during the present study.

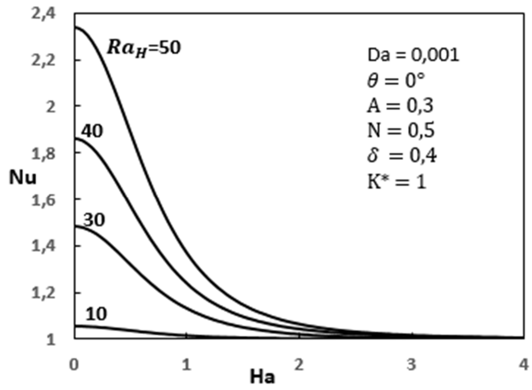

Figure 6. Effect of Ha on Nu for different Ra.

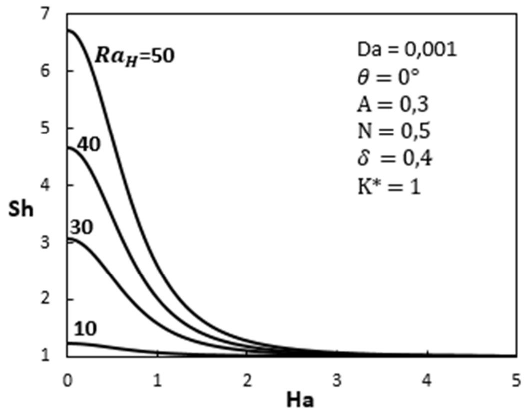

Figure 7. Effect of Ha on Sh for different Ra.

Figures 6 and 7 illustrate the variations of the heat and mass transfer rate in porous media as a function of Hartmann for different values of the permeability ratio K* for Ra = 50; Da = 0.05; θ =45°, A = 0.2; δ = 0.4 et N =0.8.

From the analysis of Figure 6, it emerges that the rate of heat transfers decreases sharply for different values of Ra as the Hartmann number increases. This sudden decrease tends to the conduction regime for which Nu = 1. Moreover, the value of Ra for which the pure conduction regime is reached depends on the Hartmann number and therefore on the transverse magnetic field (for Ra = 30; Ha = 2.5 and Ra = 50 Ha = 2.5). Moreover, from Figure 7, we also retain that the mass rate decreases sharply for different values of Ra when the Hartmann number increases. For Ha > 2.5 and Ra = 40 we have Sh = 1 and Ha > 3.5 and Ra = 50 we have Sh = 1 which presents pure conduction.

Figures 8 and 9 illustrate the variations of the heat and mass transfer rate in porous media as a function of Hartmann for different values of the permeability ratio K * when Ra= 100; Da = 0.01, A = 0.5, K* = 1, θ = 0°, δ = 0.3, N = 0.4.

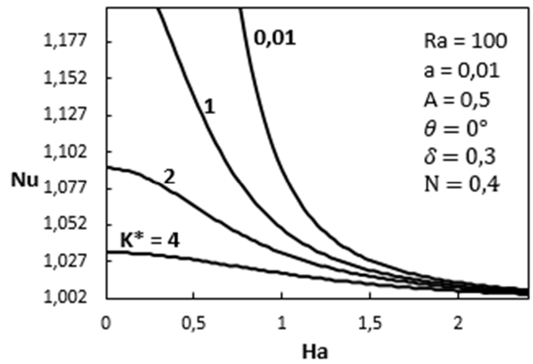

Figure 8. Effect of Ha on Nu for different K.*

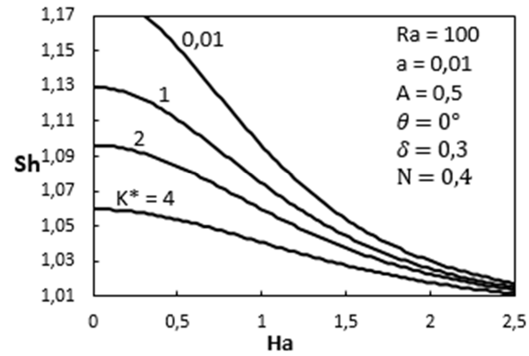

*Figure 9. Effect of Ha on Sh for different K *.*

From the analysis of Figures 8 and 9 we retain that the rate of heat and mass gradually decrease when K * increases for a low value of Ha. Likewise, when the Hartmann number increases, the Nusselt and Sherwood numbers decrease sharply for different anisotropy values, each tending towards unity (Nu = 1 and Sh = 1), which indicates the conduction regime. Figures 10 and 11 illustrate the variations of the Rayleigh number Ra on the heat and mass transfer by convection for Da= 0.05, A=0.5; K*=0.1, θ=15°, δ=0.6 et N=0.7.

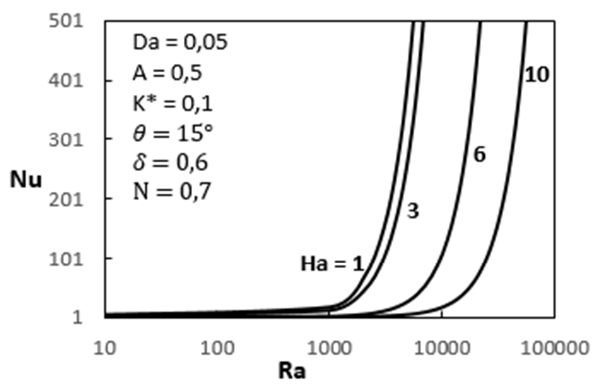

Figure 10. Effect of Ra on Nu for different Ha.

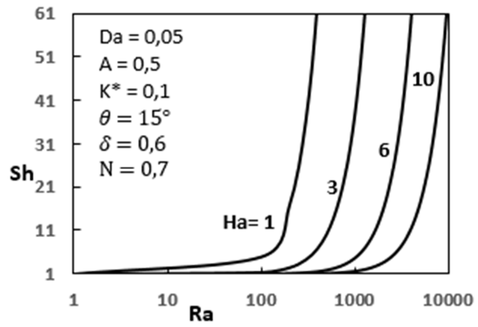

Figure 11. *Effect of Ra on Sh for different Ha.*

From the analysis of Figures 10 and 11, it emerges that the heat and mass transfers increase from the pure conduction regime. For a fixed Rayleigh number, as the Hartmann number increases the rate of heat transfer and mass decreases. Results validated by the work of Akowanou [13] for $\delta = 0$.

Figures 12 and 13 illustrate the variations of the Darcy Da number on heat and mass transfer by convection for Da= 0.05, A=0.5; K*=0.25, θ=15°, δ=0.6 and N=0.7.

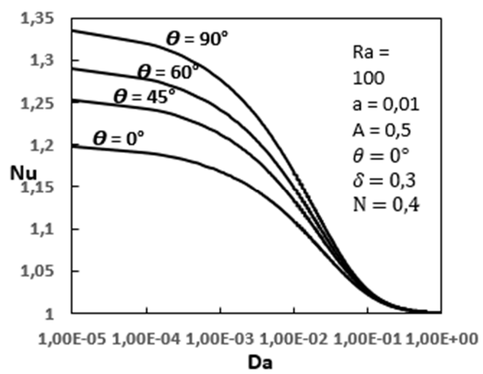

Figure 12. *Effect of Da on Sh for different θ.*

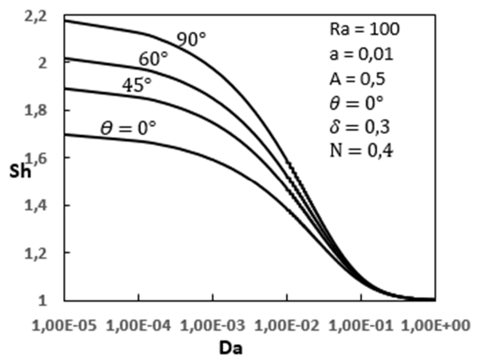

Figure 13. *Effect of Da on Sh for different θ.*

From the analysis of Figures 12 and 13, it emerges that for low Darcy values (Da≤1) the Nusselt and Sherwood number tend asymptotically towards a constant value which strongly depends on the orientation angle value. For different values of the orientation angle we deduce that the heat and mass transfer rates have each reached their maximum values for θ = 90° and their minimum values for θ = 0°. Results validated by the work of Degan 1997 [13] in Natural convection in an anisotropic porous cavity: Brinkman model, Chap 6 page 174.

8. Conclusion

A study has been made of bi-diffusive convection in a cavity filled by a porous medium saturated by a binary fluid. The cavity is submitted to variable thermal conditions on its wall. The porous medium, assumed to be hydrodynamically anisotropic with its principal axes oriented in a direction that is oblique to the gravity is submitted to transverse magnetic field effect. Analytical expressions valid for fully developed flow and based on the generalized Brinkman-extended Darcy model are obtained. The major conclusions of the present paper can be expressed as follow:

a) Both magnetic field and anisotropic parameters of the porous medium have a strong influence on the fluid motion, the heat and mass transfer through the porous matrix;
b) Heat and mass transfer in the porous medium become very important when the permeability in the horizontal direction is higher than that one prevailing in the vertical direction;
c) Increasing the applied transverse magnetic field significantly reduces the flow velocity saturating the porous medium and then attenuates the heat and mass transfer.

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