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Review Article

A Class of New Exact Solution of Equations for Motion of Variable Viscosity Fluid in Presence of Body Force with Moderate Peclet Number

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Abstract: This is to communicate a class of new exact solutions of the equations governing the steady plane motion of fluid with constant density, constant thermal conductivity but variable viscosity and body force term to the right-hand side of Navier-Stokes equations with moderate Peclet numbers. Exact solutions are obtained for Peclet numbers between zero and infinity except 2, for given one component of the body force using successive transformation technique and a new characterization for the streamlines. A temperature distribution formula, due to heat generation, is obtained when Peclet number is 4 other wise temperature distribution is found to be constant. The exact solutions are large in number as streamlines, velocity components, viscosity function, and energy function and temperature distribution in presence of body force exists for a huge number of the moderate Peclet number.

Keywords: Variable Viscosity Fluids, Moderate Peclet Number, Navier-Stokes Equations with Body Force, Incompressible Fluids

1. Introduction

Theoretical study of fluid flow problem requires deriving the equations for law of conservation of mass, momentum and energy. The momentum equations are Navier-Stokes equations (NSE). The way in which it is derived, allows us to add body force term like centrifugal force, coriolis force, constant gravity force etc. to the right-hand side of it. The introduction of body force term is important for the study of large-scale fluid flow. In the presence of unknown external force the law of conservation of mass, momentum and energy in non-dimensional form for steady flow problem of fluid with constant density, constant thermal conductivity, constant specific heat but variable viscosity using following dimensionless parameters [1]

$$x^* = \frac{x}{L_0} \quad y^* = \frac{y}{L_0} \quad u^* = \frac{u}{U_0} \quad v^* = \frac{v}{U_0}$$

$$\mu^* = \frac{\mu}{\mu_0} \quad p^* = \frac{p}{p_0} \quad F_1^* = \frac{F_1}{F_0} \quad F_2^* = \frac{F_2}{F_0}$$

in tensor notation, after removing the overhead "*", are following [2]

$$\nabla.\mathbf{v} = \frac{\partial v_k}{\partial x_k} = 0 \tag{1}$$

$$\left(v_k \frac{\partial v_i}{\partial x_k}\right) = F_i - \frac{\partial p}{\partial x_i} + \frac{1}{R_e}\frac{\partial}{\partial x_j}\left\{\mu\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right)\right\} \tag{2}$$

$$\left(v_k \frac{\partial T}{\partial x_k}\right) = \frac{1}{R_e P_r} \frac{\partial}{\partial x_i}\left(\frac{\partial T}{\partial x_i}\right) + \frac{E_c}{R_e} \mu \frac{\partial v_i}{\partial x_j}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right) \tag{3}$$

Where $\mathbf{F} = F_j(x_i)$ is the body force per unit mass, $\mathbf{v} = \left(u(x_i), v(x_i)\right)$ is the fluid velocity vector, $p = p(x_i)$ is pressure, ρ is constant fluid density and $\mu = \mu(x_i)$ is viscosity, $i, j, k \in \{1,2,3\}$. The dimensionless quantity R_e, P_r and E_c are known as the *Reynolds number*, the *Prandtl number* and the *Eckert number* respectively. The product of R_e and P_r is Peclet number denoted by $P_{e'}$. For the plane case $i, j, k \in \{1,2\}$, $x_1 = x$, $x_2 = y$, $v_1 = u$ $v_2 = v$, $\mathbf{F} = \left(F_1(x,y),\ F_2(x,y)\right)$, reduces the equations (1-3) to following

$$u_x + v_y = 0 \tag{4}$$

$$u\, u_x + v\, u_y = F_1 - p_x + \frac{1}{\mathrm{R_e}} [(2\mu u_x)_x + \{\mu(u_y + v_x)\}_y] \tag{5}$$

$$u\, v_x + v\, v_y = F_2 - p_y + \frac{1}{\mathrm{R_e}} [(2\mu v_y)_y + \{\mu(u_y + v_x)\}_x] \tag{6}$$

$$u\left(\frac{2}{a}\right) + v\, T_y = \frac{1}{P_{e'}} (T_{xx} + T_{yy}) + \frac{\mathrm{E_c}}{\mathrm{R_e}} [2\mu(u_x{}^2 + v_y{}^2) + \mu(u_y + v_x)^2] \tag{7}$$

The solution of the equation (4) is a stream function $\psi(x,y)$ such that the velocity components are

$$\mathbf{v} = (u,v) = \left(\psi_y, -\psi_x\right) \tag{8}$$

Like other mechanics, fluid dynamics also offers difficulties due to nonlinear nature of the NSE and it is extremely difficult to reach exact solutions. Therefore, theoretical scientists invent techniques to achieve exact solutions of basic equations of fluid dynamics. Those who are interested in solution techniques for a given Peclet number please refer to [3-5] and reference therein. Theorists have also devised some coordinates transformation techniques and dimension analysis methods for exact solutions to these equations. Readers interested in these methods/techniques may refer to [1-2, 6-9] and the references therein.

This discourse determines a class of exact solutions to flow equations (5-7) using method of partial differentiation and successive transformation technique as applied in [1-2] and [6-8]. Firstly, the equations (5-7) are rewritten, through the partial differentiation technique, in terms of the vorticity function w and the total energy function T_x, the function A and B defined as follow

$$w = v_x - u_y \tag{9}$$

$$T_x = p + \frac{1}{2}(u^2 + v^2) - \frac{1}{\mathrm{R_e}} y_\psi \tag{10}$$

$$A = \mu(u_y + v_x) \quad B = 4\mu\, u_x \tag{11}$$

Secondly, the resulting basic flow equations are transformed into curvilinear coordinate system (ϕ,ψ) through transformation

$$x = x(\phi,\psi),\ y = y(\phi,\psi) \tag{12}$$

in terms of the coefficients of the first fundamental form

$$ds^2 = E(\phi,\psi)\, d\phi^2 + 2F(\phi,\psi)\, d\phi\, d\psi + G(\phi,\psi)\, d\psi^2 \tag{13}$$

wherein

$$E = x_\phi^2 + y_\phi^2,\ F = x_\phi\, x_\psi + y_\phi\, y_\psi,\ G = (x_\psi)^2 + (y_\psi)^2 \tag{14}$$

such that the Jacobian $J = \dfrac{\partial(x,y)}{\partial(\phi,\psi)}$ of the transformation is non-zero and finite.

Martin [10] defined the coordinate lines $\psi = \text{constant}$ as streamlines and left the coordinate lines $\phi = \text{constant}$ arbitrary in a study of fluid flow problem therefore the curvilinear coordinate (ϕ,ψ). These curvilinear coordinates system will be referred here as Martin's coordinates system (ϕ,ψ). Let λ be the angle between the tangent to the coordinate lines $\psi = const.$ and the curves $\phi = const.$ at a point $P(x,y)$ then it is easy to write the equations (4-7), the function A, B and w into Martin's system applying differential geometric technique. Let us skip these equations as they are already published in [1]. The arbitrary coordinate lines $\phi = \text{constant}$ of Martin's coordinates system (ϕ, ψ) can be defined in many ways. For example in the von-Mises coordinates system (x, ψ), the coordinate lines $\phi = \text{constant}$ is taken along $x - axis$ i.e. choosing function $\phi = x$ and stream function ψ of Martin's coordinates as independent variables instead of y and x [11]. Similarly, the coordinates system (r, ψ) considers the coordinate lines $\phi = \text{constant}$ as $\phi = r(x, y)$ and stream function ψ of Martin's coordinates as independent variables instead of y and x [1-2, 6-8]. Rana Khalid Naeem, a Pakistani Mathematician, suggested the coordinates system (r, ψ) for the study [2] and unfortunately expired just after the publication [6] about the exact solutions of NSE in the absence of body force. This communication, intending the exact solutions of NSE in the presence of body force, is also focusing on the coordinates system (r, ψ), therefore it sets the coordinate lines $\phi = \text{constant}$ of Martin's system, as follow

$$\phi = r(x, y) \tag{15}$$

where

$$x = r\cos\theta \quad y = r\sin\theta \tag{16}$$

Let the characterization of streamlines is through the equation

$$\frac{\theta - f(r)}{g(r)} = \nu(\psi) \tag{17}$$

where $f(r)$ and $g(r) \neq 0$ are continuously differentiable functions and r, θ the polar coordinates. The equation (17), without loss of generality, implies

$$\theta = f(r) + g(r)\nu(\psi) \tag{18}$$

When the streamlines are characterized by equation (18), exact solutions in presence of body force are obtained for the case when both $g'(r) = 0$ and $\nu''(\psi) = 0$ keeping the function $f(r)$ arbitrary in [1]. In [7], exact solutions in presence of body force are obtained for the case when $g'(r) = 0$ but $\nu''(\psi) \neq 0$ keeping the function $f(r)$ arbitrary. Similarly, [8] used the functions $g'(r) \neq 0$ and $\nu''(\psi) = 0$ in order to find exact solutions in presence of body force. The aim in this communication is to achieve exact solutions in presence of body force taking $g'(r) \neq 0$ and $\nu''(\psi) \neq 0$ for a function $f(r)$.

Utilizing (15-16), the basic flow equations are transform from Martin's coordinates system to the coordinates system (r, ψ), the vorticity function w, the function A and B are following [1-2]

$$-R_e\, w = -R_e\, J\, F_2 + R_e\, L_\psi - JA_r + \sqrt{E-1}\, A_\psi + B_\psi \tag{19}$$

$$0 = R_e\left(F_1 + F_2\sqrt{E-1}\right) - R_e\, L_r + \frac{A_\psi(2-E)}{J} + A_r\sqrt{E-1} - \frac{\sqrt{E-1}\, B_\psi}{J} \tag{20}$$

$$J\,T_{rr} - 2\sqrt{E-1}\;\; T_{\nu r}\,\nu' + \frac{E}{J}T_{\nu\nu}\,(\nu')^2 + \left(J_r - \frac{E_\psi}{2\sqrt{E-1}} - P_{e'}\right)T_r$$

$$+\left(\frac{E_\psi}{J} - \frac{E_r}{2\sqrt{E-1}} + \frac{EJ_\psi}{J^2} + \frac{E}{J}\left(\frac{\nu''}{\nu'}\right)\right)T_\nu \nu' = -\frac{J\,E_c\,P_r}{4\mu}\left(B^2 + 4A^2\right) \tag{21}$$

$$w = \left[\frac{(rf'' + f')}{rg} - \frac{2f'g'}{g^2}\right]\left(\frac{1}{\nu'}\right) + \left[\frac{(rg'' + g')}{rg} - \frac{2(g')^2}{g^2}\right]\left(\frac{\nu}{\nu'}\right) + \left(\frac{1}{r^2 g^2} + \frac{f'^2}{g^2}\right)\left(\frac{\nu''}{(\nu')^3}\right) + \left(\frac{2f'g'}{g^2}\right)\left(\frac{\nu\nu''}{(\nu')^3}\right) + \frac{g'^2}{g^2}\left(\frac{\nu^2\nu''}{(\nu')^3}\right) \tag{22}$$

$$A = \frac{\mu}{r^2 g^2 \nu'}\left[rg\left[(rf'' + f') + (rg'' + g')\nu\right] - \left\{1 - r^2(f' + g'\nu)^2\right\}\left(\frac{\nu''}{\nu'^2}\right) - 2r(f' + g'\nu)(rg' + g)\right] \tag{23}$$

$$B = \frac{4\mu}{r^2 g^2 \nu'}\left[-(rg' + g) + r(f' + g'\nu)\left(\frac{\nu''}{\nu'^2}\right)\right] \tag{24}$$

$$E = 1 + r^2\left[f'(r) + g'(r)\nu(\psi)\right]^2 \tag{25}$$

$$J = rg(r)\nu'(\psi) \tag{26}$$

where the non-zero body force $\mathbf{F} = \left(F_1(r,\psi), F_2(r,\psi)\right)$ in (r, ψ) coordinates.

Differentiating equation (19) with respect to r, differentiating equation (20) with respect to ψ and the function L satisfying the natural condition $L_{r\psi} = L_{\psi r}$, following additional compatibility equation is constructed

$$JA_{rr} - 2\sqrt{E-1}\,A_{r\psi} - \frac{(2-E)}{J}A_{\psi\psi} + A_r\left(J_r - \frac{E_\psi}{2\sqrt{E-1}}\right) + A_\psi\left(-\frac{E_r}{2\sqrt{E-1}} + \frac{J_\psi(2-E)}{J^2} + \frac{E_\psi}{J}\right) - \left\{B_r - \frac{\sqrt{E-1}\,B_\psi}{J}\right\}_\psi = R_e w_r + R_e\left(F_1 + F_2\sqrt{E-1}\right)_\psi - R_e\left(JF_2\right)_r \tag{27}$$

2. Exact Solution

The compatibility equation (27) involves functions A and B which depends upon the viscosity function μ, $f(r)$, $g(r)$ and $\nu(\psi)$ which in general is difficult to solve analytically. The elimination of μ from the function A and B found some time be helpful in solving the compatibility equation, see [1, 6-7], and some time it may not see [8]. Luckily a relation between the function A and B is found feasible for the class of flows under consideration. As the case under consideration requires $\nu''(\psi) \neq 0$ and $g'(r) \neq 0$ therefore setting

$$\nu(\psi) = e^\psi \tag{28}$$

$$g = \frac{c}{r} \tag{29}$$

in equations (22-24) leads to

$$f(r) = \ln r + b \tag{30}$$

where c and b are constant. Equations (22-24) on utilization equations (28-30) provides

$$w = \left(\frac{2}{c^2}\right)\left(\frac{1}{(e^\psi)^2}\right) \tag{31}$$

$$A = -\frac{\mu}{c^2 e^\psi}\frac{2c}{r}\left(1 - \frac{ce^\psi}{r}\right) \tag{32}$$

and

$$B = \frac{4\mu}{c^2 e^\psi}\left(1 - \frac{ce^\psi}{r}\right)\left(\frac{1}{e^\psi}\right) \tag{33}$$

It is easy to eliminate the viscosity function μ from equation (32) and equation (33) and find following relation between the function A and B

$$B = \frac{-2r}{ce^\psi}A \tag{34}$$

Substituting equation (28-30) and equation (34), the equation (27) reduces to

$$ce^\psi A_{rr} - 2\left(1 - \frac{ce^\psi}{r} - \frac{re^{-\psi}}{c}\right)A_{r\psi} + \left(-\frac{2}{r} + \frac{ce^\psi}{r^2} + \frac{2e^{-\psi}}{c} - \frac{2re^{-2\psi}}{c^2}\right)A_{\psi\psi} + \left(\frac{ce^\psi}{r} - \frac{2re^{-\psi}}{c}\right)A_r + \left[\frac{6re^{-2\psi}}{c^2} - \frac{2e^{-\psi}}{c}\right]A_\psi - \frac{4re^{-2\psi}}{c^2}A = R_e\left(F_1\right)_\psi + R_e\left((1 - \frac{ce^\psi}{r})F_2\right)_\psi - R_e\left(ce^\psi F_2\right)_r \tag{35}$$

The equation (35) involves three functions A, F_1 and F_2 in two independent variables r and ψ therefore, setting

$$R_e\left((1 - \frac{ce^\psi}{r})F_2\right)_\psi - R_e\left(ce^\psi F_2\right)_r = 0 \tag{36}$$

implies

$$R_e F_2 = \frac{R_e}{r}Q\left(\frac{e^\psi}{r} + \frac{1}{c}\ln r\right) \tag{37}$$

for arbitrary function $Q\left(\frac{e^\psi}{r} + \frac{1}{c}\ln r\right)$. Equation (37) on substituting in equation (35) gives

$$R_e\left(F_1\right)_\psi = c\,e^\psi A_{rr} - 2\left(1-\frac{ce^\psi}{r}-\frac{re^{-\psi}}{c}\right)A_{r\psi} + \left(-\frac{2}{r}+\frac{ce^\psi}{r^2}+\frac{2e^{-\psi}}{c}-\frac{2re^{-2\psi}}{c^2}\right)A_{\psi\psi}$$

$$+\left(\frac{ce^\psi}{r}-\frac{2re^{-\psi}}{c}\right)A_r + \left[\frac{6re^{-2\psi}}{c^2}-\frac{2e^{-\psi}}{c}\right]A_\psi - \frac{4re^{-2\psi}}{c^2}A \tag{38}$$

The following selection of the form of the function A is found helpful in finding the function F_1 from equation (38)

$$A(r,\psi) = R(r)\,S(\psi) \tag{39}$$

The equation (38) on substituting (39) gives

$$R_eF_1 = c\,R''\int e^\psi\,S d\psi - \frac{cR'}{r}\int e^\psi\,S + \frac{cR}{r^2}\int e^\psi\,S + \left(-2+\frac{2c}{r}e^\psi+\frac{2r}{c}e^{-\psi}\right)R'S$$

$$+\left(-\frac{2}{r}-\frac{c}{r^2}e^\psi+\frac{2r}{c^2}e^{-2\psi}\right)RS + \left(\frac{c}{r^2}e^\psi+\frac{2}{c}e^{-\psi}-\frac{2r}{c^2}e^{-2\psi}\right)RS' + K_1(r) \tag{40}$$

where $K_1(r)$ is a function of integration. Defining the functions F_1 and F_2 by equation (40) and equation (37) respectively the compatibility equation (27) is satisfied.

The viscosity from either equation (32) or equation (33) is following

$$\mu = -\frac{ce^\psi\,r}{2}\left(1-\frac{ce^\psi}{r}\right)^{-1} R(r)\,S(\psi) \tag{41}$$

The function L satisfying both the momentum equations (19-20) is

$$R_e\,L = \left(\frac{R_e}{c^2}\right)e^{-2\psi} - \frac{2b_1r}{c}R(r)\,e^{-\psi} + \left(\frac{R_e\ln r}{r}-b_1cR'(r)\right)e^\psi + \frac{cR_ee^{2\psi}}{2r^2}$$

$$+b_1\,R + 2\,b_1\int\frac{R(r)}{r}dr + \frac{R_e\left(\ln r\right)^2}{2c} + b_2 + \int K_1(r)dr \tag{42}$$

where b_1 and b_2 are constants and the compatibility of momentum equation requires the arbitrary function Q to be a linear function that is

$$Q\left(\frac{e^\psi}{r}+\frac{1}{c}\ln r\right) = \left(\frac{e^\psi}{r}+\frac{1}{c}\ln r\right) \tag{43}$$

Equation (21) on utilizing equation (28-33) simplifies to

$$c\,e^\psi\,T_{rr} - 2\left(1-\frac{c\,e^\psi}{r}\right)T_{\psi r} + \left(\frac{2e^{-\psi}}{c}-\frac{2}{r}+\frac{ce^\psi}{r^2}\right)T_{\psi\psi} + \left(\frac{c\,e^\psi}{r}-P_{e'}\right)T_r$$

$$-\left(\frac{2e^{-\psi}}{c}\right)T_\psi = -2\,b_1\,E_c\,P_r\left(\frac{re^{-2\psi}}{c^2}-\frac{e^{-\psi}}{c}+\frac{1}{r}-\frac{c\,e^\psi}{r^2}\right)R(r) \tag{44}$$

Right hand side of equation (44) suggests searching for a solution of the form

$$T(r,\psi) = T_1(r) + T_2(r)\,e^\psi + T_3(r)\,e^{-\psi} + T_4(r)\,e^{-2\psi} \tag{45}$$

Equation (44) on substituting equation (45) reduces to

$$\frac{12T_4\,e^{-3\psi}}{c} + \left[4T_4' - \frac{8T_4}{r} - P_{e'}\,T_4' + \frac{4T_3}{c}\right]e^{-2\psi} + \left[c\,T_4'' - \frac{4c\,T_4'}{r} + \frac{4cT_4}{r^2} + \frac{c\,T_4'}{r} + (2-P_{e'})T_3' - \frac{2T_3\,e^{-\psi}}{r}\right]e^{-\psi}$$

$$+cT_3'' - \frac{cT_3'}{r} + \frac{cT_3}{r^2} - P_{e'}T_1' + \left[cT_1'' - (2+P_{e'})T_2' - \frac{2T_2}{r} + \frac{cT_1'}{r}\right]e^{\psi}$$

$$+\left[cT_2'' + \frac{3cT_2'}{r} + \frac{cT_2}{r^2}\right]e^{2\psi} = -2b_1 E_c P_r \left(\frac{1}{r} - \frac{ce^{\psi}}{r^2} - \frac{e^{-\psi}}{c} + \frac{re^{-2\psi}}{c^2}\right)R(r) \quad (46)$$

Equation (46) on comparing the like coefficients on both side and solving the resulting equations simultaneously concludes to

$$T_4 = 0 \quad (47)$$

$$R = t_0 r^{\alpha} \text{ where } \alpha = \frac{(-4+P_{e'})}{(2-P_{e'})}, \text{ when } P_{e'} \neq 2 \quad (48)$$

$$T_1 = \frac{E_c b_1}{2R_e}\left\{-rR' + 2\int\left(\frac{R}{r}\right)dr\right\} + b_3 \quad (49)$$

$$T_2 = \frac{b_4}{r} \quad (50)$$

$$T_3 = -\frac{E_c P_r b_1}{2c} rR \quad (51)$$

where b_3, b_4 and t_0 are arbitrary constants and

$$P_{e'}C_1 - \frac{cE_c m_1 t_0}{2R_e} r^{\alpha}\left[\alpha^3 - 4\alpha + 4P_{e'}\right] = 0 \quad (52)$$

The solution equation (52) will be discussed on the choice of α, when α is zero and when it is not.

For the case $\alpha = 0$ (or $P_{e'} = 4$) the equations (47-51) simplifies to

$$R = t_0 r^{\alpha} = t_0, \ b_4 = \frac{2cE_c b_1 t_0}{R_e}, \ T_1(r) = \frac{2E_c b_1 t_0 \ln r}{R_e} + b_3,$$

$$T_2(r) = \frac{2cE_c b_1 t_0}{R_e}\frac{1}{r}, \ T_3 = -\frac{E_c P_r b_1 t_0}{2c} r \text{ and } T_4 = 0$$

and from equation (45) the temperature formula due to heat generation is

$$T = b_3 + \frac{E_c b_1 t_0 P_r}{2}\left\{\ln r + \frac{ce^{\psi}}{r} - \frac{r}{ce^{\psi}}\right\} \quad (53)$$

For the case $\alpha \neq 0$ (or $P_{e'} \neq 4$) the equations (38-42) simplifies to

$b_4 = 0$, $b_1 = 0$, $R = t_0 r^{\alpha}$,

$T_1(r) = b_3$, $T_2(r) = 0$, $T_3 = 0$, and $T_4 = 0$

and the temperature due to heat generation is constant as it is obvious from equation (45)

$$T = b_2 \quad (54)$$

Thus viscosity from (41), pressure from (10) using (42) and velocity from equation (8) for $P_{e'} \in (0, \ \infty)$ except $P_{e'} = 2$ and temperature form equation (53) when $P_{e'} = 4$ and from (54) when $P_{e'} \neq 4$ is obtained.

3. Conclusions

In this communication, a class of new exact solutions of the equations governing the steady plane motion of fluid of constant density, constant thermal conductivity but variable viscosity in presence of body force term for moderate Peclet numbers $P_{e'} \in (0, \ \infty)$ except $P_{e'} = 2$ for given one component of the body force is obtained. The streamline $\psi = const.$ for class of flows under consideration is found to be $\theta = b + \ln r + \dfrac{c\, e^{\psi}}{r}$ in polar coordinates r , θ . A temperature distribution formula $T = b_3 + \dfrac{E_c b_1 t_0 P_r}{2}\left\{\ln r + \dfrac{c\, e^{\psi}}{r} - \dfrac{r}{c\, e^{\psi}}\right\}$ is identified, due to heat generation, when $P_{e'} = 4$ otherwise the temperature due to heat generation found to remain constant. As, streamlines, velocity components, viscosity function, and the energy function and temperature distribution in the presence of body force for all moderate Peclet number can be constructed therefore this shows a large number of exact solutions to the flow problem.

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